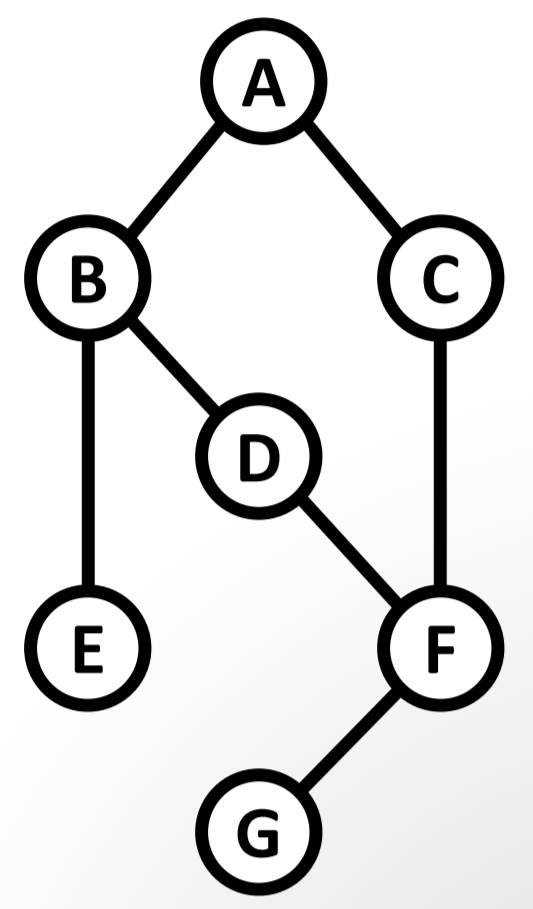


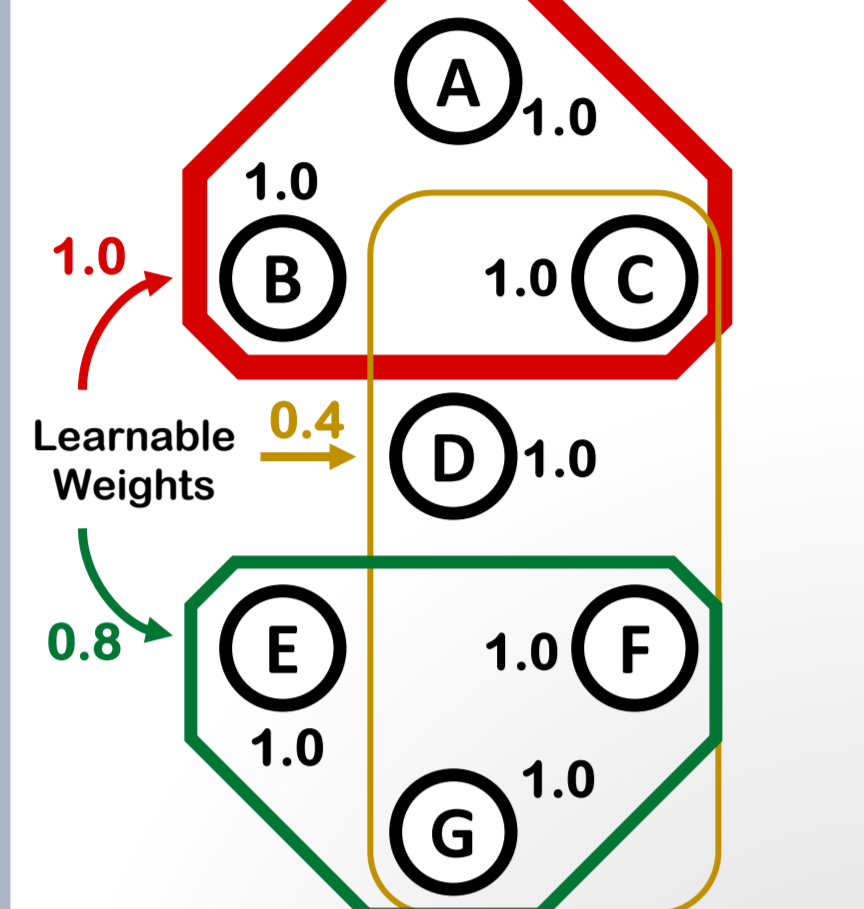
INTRODUCTION

Problem 1. Pairwise Limitation



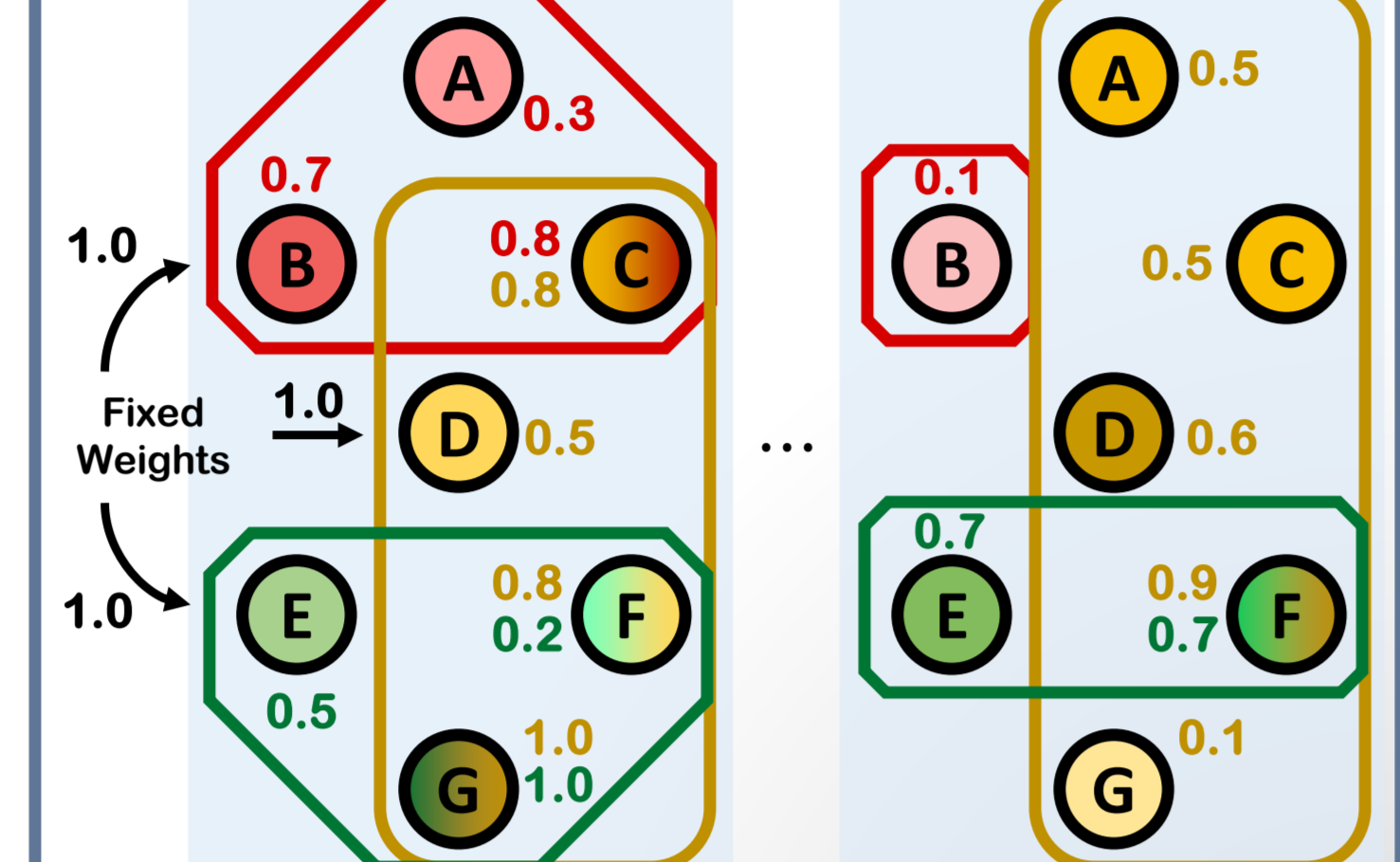
Pair-wise Relations Only
Missing Group Interactions
Fixed Node Contributions

Problem 2. Restricted Hypergraph Learning



Predefined Hyperedges
Hyperedge-level Weights
Uniform Intra-Group Nodes

Our Approach: Multi-Scale Hypergraph Learning



Adaptive Hyperedge Discovery
Node-level Group Activations
Multi-scale High-order Patterns

MUHL

Problem:

1. Pairwise Limitation: Existing graph-based brain network models mainly capture pairwise ROI connections, limiting their ability to model group-wise interactions among multiple brain regions.

2. Restricted Hypergraph Learning: Existing hypergraph-based methods often rely on predefined hyperedges or only adjust hyperedge-level weights, without fully learning node-level activation patterns within each hyperedge.

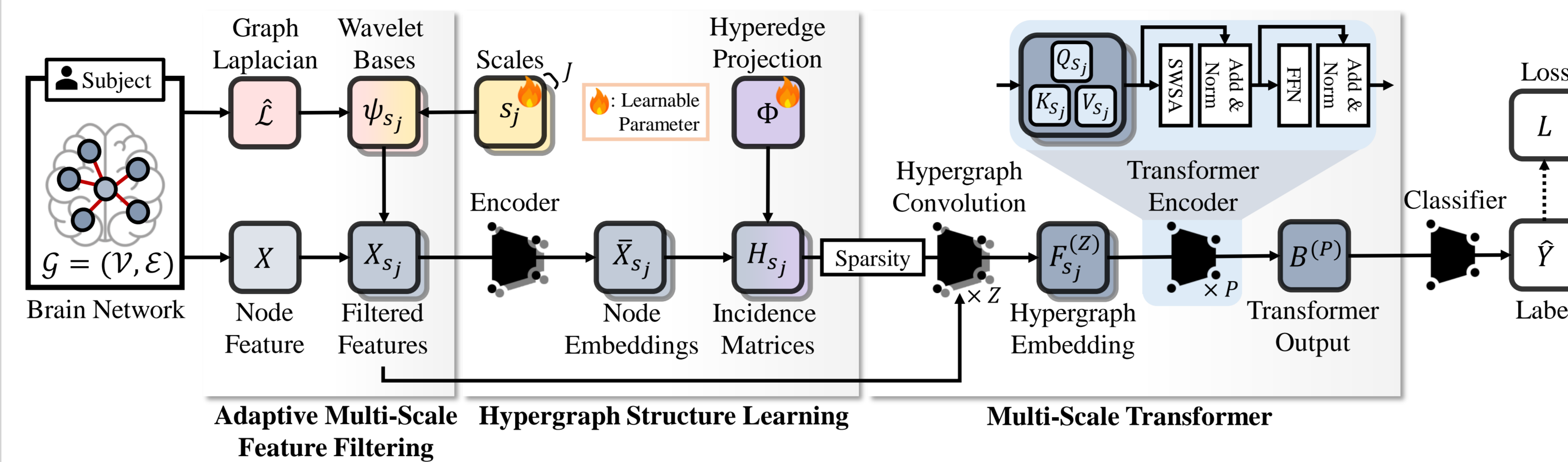
Contribution:

1. Dynamic Hyperedge Structure Learning: *MuHL* dynamically learns rich and implicit group-wise node dependencies (i.e., hyperedges).

2. Adaptive Multi-Resolution Brain Representations: *MuHL* adaptively captures structural variations at different scales for hypergraph construction to enhance downstream classification for brain network analysis.

3. Theoretical Justification: *MuHL* demonstrates theoretical insights that the adaptive hyperedges are derived by modulating node features.

MULTI-SCALE HYPERGRAPH LEARNING (MuHL)



(1) Adaptive Multi-Scale Feature Filtering

- Extract scale-specific ROI features using learnable graph wavelet scales.

$$X_{s_j} = U g^2(s_j \Lambda) U^T X, \quad j = 1, \dots, J$$

- where X is the ROI feature matrix, U and Λ are the eigenvectors and eigenvalues of the graph Laplacian, $g(\cdot)$ is a spectral wavelet kernel, and s_j is a learnable scale parameter.

(2) Hypergraph Structure Learning

- Construct adaptive hyperedges by projecting node embeddings into hyperedge space.

$$\tilde{H}_{s_j} = \text{Softmax}(\text{ReLU}(\tilde{X}_{s_j} \Phi)), \quad \tilde{H}_{s_j} = \text{TopK}_{\eta}(\tilde{H}_{s_j})$$

- where \tilde{X}_{s_j} denotes the embedded node representation, Φ is a learnable hyperedge projector, \tilde{H}_{s_j} is a soft incidence matrix, and \tilde{H}_{s_j} is its sparse version with at most η hyperedges per node.

(3) Multi-Scale Transformer

- Aggregate scale-wise hypergraph features for graph-level disease classification.

$$F_{s_j} = \text{HGConv}(X_{s_j}, \tilde{H}_{s_j}), \quad B = \text{SWSA}(\{F_{s_j}\}_{j=1}^J)$$

- where F_{s_j} is the scale-wise hypergraph embedding from hypergraph convolution, and B is the final multi-scale representation produced by scale-wise self-attention for graph classification.

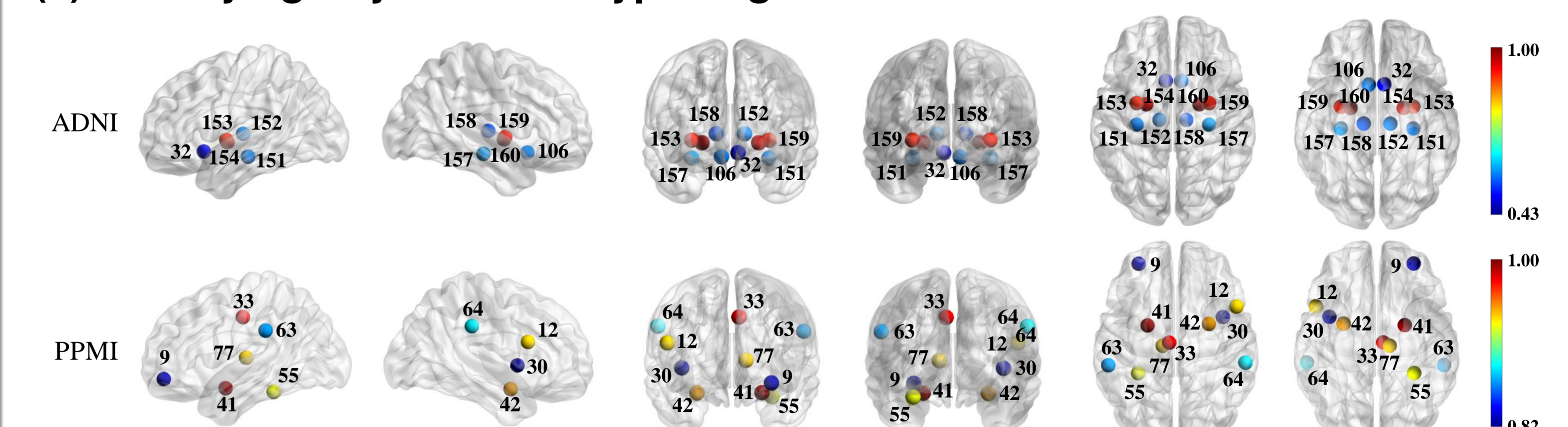
EXPERIMENTS RESULT

(2) Zero-Shot Classification Performance

Table: Zero-shot classification performance across datasets and acquisition phases. MuHL and baselines are trained on a source dataset or phase and directly evaluated on unseen target datasets or phases without fine-tuning.

Method	ADNI-2→ADNI-1/3/GO				PPMI→TaoWu				PPMI→Neurocon			
	Acc ↑	Pre ↑	Rec ↑	F1s ↑	Acc ↑	Pre ↑	Rec ↑	F1s ↑	Acc ↑	Pre ↑	Rec ↑	F1s ↑
BrainGB	42.9 ± 3.2	48.4 ± 8.1	36.5 ± 2.4	34.4 ± 4.3	53.0 ± 1.2	64.1 ± 6.8	53.0 ± 1.2	43.5 ± 4.0	62.9 ± 6.4	52.5 ± 2.8	51.6 ± 7.3	45.1 ± 5.3
BrainNetTF	43.5 ± 8.4	48.9 ± 2.4	50.8 ± 2.8	44.8 ± 1.6	50.0 ± 1.6	46.0 ± 3.9	50.0 ± 1.6	46.2 ± 4.6	62.9 ± 4.4	62.7 ± 8.1	55.0 ± 5.6	50.3 ± 8.7
ALTER	43.8 ± 3.0	46.8 ± 2.0	54.6 ± 6.4	44.3 ± 2.3	54.5 ± 1.9	65.1 ± 6.0	52.5 ± 1.9	45.3 ± 4.4	63.8 ± 3.2	63.8 ± 7.6	55.3 ± 4.4	51.7 ± 8.0
BQN	52.3 ± 2.9	39.5 ± 6.3	32.8 ± 7.7	39.0 ± 7.6	57.0 ± 4.0	56.3 ± 4.6	55.0 ± 4.0	49.5 ± 3.0	55.1 ± 3.7	47.8 ± 3.2	48.3 ± 2.9	46.8 ± 2.9
dwHGNC	52.8 ± 2.9	48.0 ± 2.7	57.0 ± 2.8	48.8 ± 3.1	50.5 ± 2.9	45.1 ± 5.2	50.5 ± 2.9	36.0 ± 4.5	60.9 ± 2.7	59.3 ± 3.4	53.6 ± 3.7	46.1 ± 7.0
HyBRID	47.6 ± 5.1	46.7 ± 4.6	53.3 ± 2.9	44.0 ± 5.2	56.2 ± 3.1	53.8 ± 3.5	49.8 ± 4.7	44.7 ± 3.8	63.4 ± 2.4	51.7 ± 2.5	50.0 ± 4.2	48.8 ± 3.6
MuHL	56.0 ± 1.6	52.1 ± 6.1	58.4 ± 8.3	54.8 ± 7.7	60.5 ± 1.6	72.8 ± 2.4	60.5 ± 1.6	58.9 ± 3.0	65.9 ± 3.4	68.1 ± 3.3	57.3 ± 3.2	53.7 ± 3.6

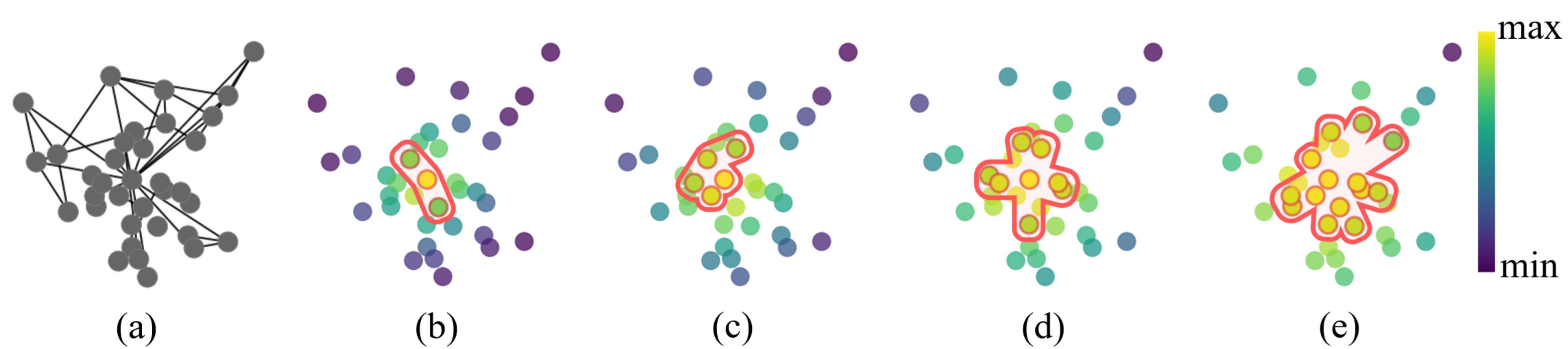
(3) Identifying Key ROIs via Hyperedge Activations



ADNI				PPMI				
Idx	ROI	Act.	Idx	ROI	Act.	Idx	ROI	Act.
154	(L) Globus.Pallidus	1.000	160	(R) Globus.Pallidus	0.963	41	(L) Amygdala	1.000
159	(R) Putamen	0.944	153	(L) Putamen	0.941	42	(R) Amygdala	0.948
157	(R) Hippocampus	0.551	151	(L) Hippocampus	0.532	12	(R) G.Frontal.Inf.Oper	0.939
106	(R) G.Subcallosal.Area	0.529	152	(L) Thalamus	0.528	64	(R) G.Supramarginal	0.888
158	(R) Thalamus	0.499	32	(L) G.Subcallosal.Area	0.432	9	(L) G.Frontal.Mid.Orb	0.829
						30	(R) Insula	0.829

Figure: Top: Visualization of top-10 ROIs with the highest relative hyperedge importance on the ADNI (top) and PPMI (bottom) datasets. (a)/(b): outer views of left/right hemisphere, (c)/(d): front/rear views, (e)/(f): top/bottom views. Node color reflects the relative contribution of each ROI based on aggregated hyperedge weights. Bottom: ROIs with 10 highest relative importance of aggregated hyperedge activation (Act.) on for classification. (L) and (R) denote the left and right hemispheres, respectively.

MULTI-RESOLUTION HIGH-ORDER RELATION MODELING



The original graph with pairwise edges is shown in (a), and from **fine scale** (b) to **coarse scale** (e), hyperedges adaptively expand to include more nodes, moving from **localized connections** to **broader groupings**.

Similar colors indicate potentially similar regions, while the red contour indicates the sparsely constructed hyperedge at each scale, determined by corresponding smoothed features.

EXPERIMENTS RESULT

(1) Brain Network Classification Performance

Table: Brain network classification performance on the ADNI and PPMI datasets evaluated using 5-fold cross-validation. The best results are shown in **bold**, and the second-best results are underlined. (†: Methods for brain network analysis)

Category	Method	ADNI (CN vs SMC vs EMCI vs LMCI vs AD)				PPMI (CN vs Prodromal vs PD)			
		Acc ↑	Pre ↑	Rec ↑	F1s ↑	Acc ↑	Pre ↑	Rec ↑	F1s ↑
Graph based method	GCN	85.7 ± 1.7	88.1 ± 3.0	82.7 ± 5.9	83.7 ± 3.4	66.3 ± 2.4	46.2 ± 5.4	41.5 ± 3.7	38.9 ± 4.4
	GAT	89.2 ± 2.1	92.7 ± 1.8	86.6 ± 4.6	88.9 ± 3.6	72.9 ± 2.5	55.1 ± 7.8	52.0 ± 3.3	52.3 ± 4.3
	GCNII	86.3 ± 4.0	91.2 ± 1.5	85.8 ± 6.6	87.4 ± 4.8	68.5 ± 2.3	52.5 ± 6.0	42.8 ± 4.5	41.4 ± 6.3
	BrainGNN†	43.4 ± 4.7	31.8 ± 3.4	28.3 ± 1.7	27.0 ± 1.3	68.1 ± 2.0	48.8 ± 5.3	40.8 ± 4.4	38.2 ± 5.3
	IBGNN†	81.4 ± 2.4	85.8 ± 0.7	82.3 ± 3.4	83.3 ± 2.3	68.1 ± 2.4	51.1 ± 6.0	42.1 ± 5.2	38.7 ± 5.4
	BrainGB†	83.5 ± 1.6	87.1 ± 2.6	80.7 ± 3.2	82.2 ± 2.3	68.8 ± 3.3	48.8 ± 6.0	42.8 ± 3.2	41.6 ± 4.5
	BrainNetTF†	89.4 ± 2.5	88.9 ± 5.7	84.6 ± 7.3	85.8 ± 6.0	67.9 ± 2.9	55.6 ± 8.8	45.7 ± 5.1	45.0 ± 7.2
	SGCN†	84.0 ± 2.1	87.6 ± 3.3	82.1 ± 4.6	83.8 ± 3.5	67.6 ± 3.2	39.0 ± 9.3	40.6 ± 4.4	37.6 ± 6.8
	ALTER†	90.8 ± 2.6	93.3 ± 2.4	89.6 ± 5.6	90.9 ± 4.6	69.6 ± 2.3	55.2 ± 8.5	55.8 ± 5.3	54.5 ± 8.3
	BioBGT†	80.3 ± 1.7	79.7 ± 7.1	77.3 ± 6.7	77.2 ± 6.1	68.0 ± 2.8	54.3 ± 8.2	48.7 ± 4.2	45.7 ± 3.2
	BQN†	81.5 ± 4.7	74.3 ± 5.6	72.3 ± 5.5	72.6 ± 5.2	71.9 ± 2.9	63.2 ± 6.2	51.0 ± 8.4	56.4 ± 7.3
	HGN†	83.2 ± 2.7	84.8 ± 7.1	80.7 ± 6.8	82.1 ± 6.6	65.2 ± 4.8	43.3 ± 9.0	41.9 ± 6.1	40.3 ± 7.1
HNNH	87.3 ± 1.8	89.6 ± 1.0	85.6 ± 4.2	87.0 ± 3.0	69.1 ± 2.9	52.6 ± 5.1	48.5 ± 8.4	48.0 ± 9.4	
UniGCNII	89.6 ± 1.6	91.1 ± 1.8	88.0 ± 3.8	89.9 ± 2.0	70.8 ± 3.3	59.6 ± 9.4	54.3 ± 7.2	55.2 ± 7.3	
HyperDrop	72.3 ± 0.8	74.4 ± 3.1	65.9 ± 0.2	68.6 ± 2.2	67.5 ± 4.0	41.5 ± 8.9	39.2 ± 3.8	36.0 ± 6.4	
Hypergraph based method	dwHGNC†	90.2 ± 1.3	87.2 ± 7.1	86.3 ± 6.8	86.2 ± 6.4	69.0 ± 2.9	51.5 ± 7.9	44.2 ± 6.1	43.3 ± 8.0
	HyperGT	81.5 ± 1.6	92.1 ± 1.8	89.0 ± 3.8	89.9 ± 2.0	68.5 ± 1.5	51.7 ± 4.4	42.0 ± 4.8	39.6 ± 6.1
	HyBRID†	86.6 ± 4.2	87.5 ± 3.9	84.3 ± 6.4	84.6 ± 4.7	67.2 ± 6.5	54.8 ± 6.8	52.0 ± 6.2	52.8 ± 6.5
	DHNN	81.1 ± 2.5	89.0 ± 4.5	84.5 ± 3.2	86.7 ± 3.4	60.6 ± 3.8	62.3 ± 9.8	52.7 ± 4.3	54.0 ± 6.3
	MuHL (Ours)	93.2 ± 2.4	95.4 ± 1.3	94.2 ± 1.6	94.7 ± 1.5	76.8 ± 3.7	66.6 ± 7.6	60.7 ± 6.0	62.4 ± 6.6

(4) Interpreting Most Salient Hyperedge

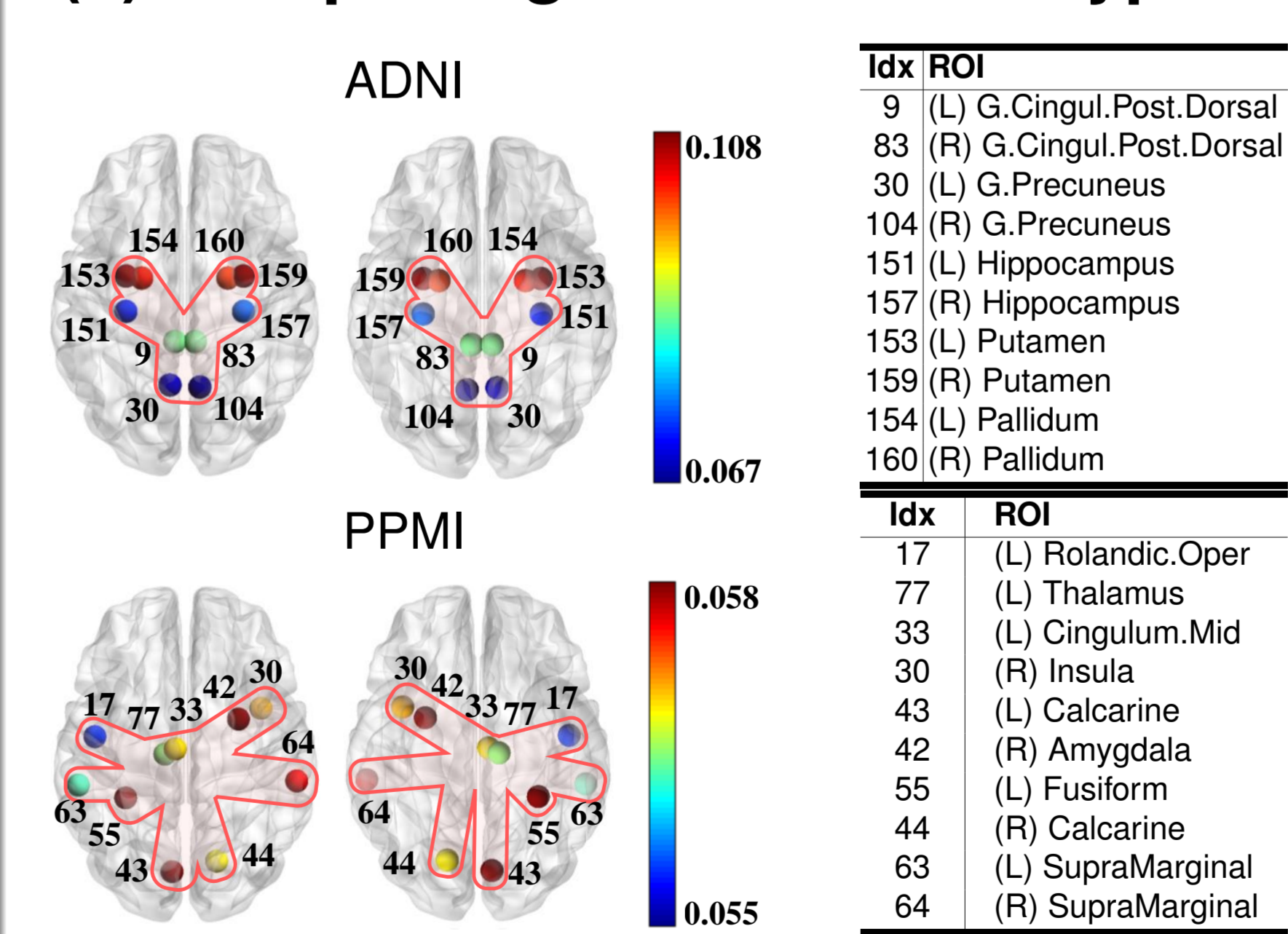


Figure: Left: Visualization of the top-10 ROIs associated with the most important hyperedge for top/bottom views, based on the sum of node activations for each hyperedge. Node color indicates activation value (i.e., weight) per node within the hyperedge. Right: Corresponding ROI labels.

(5) Hyper-parameter Studies

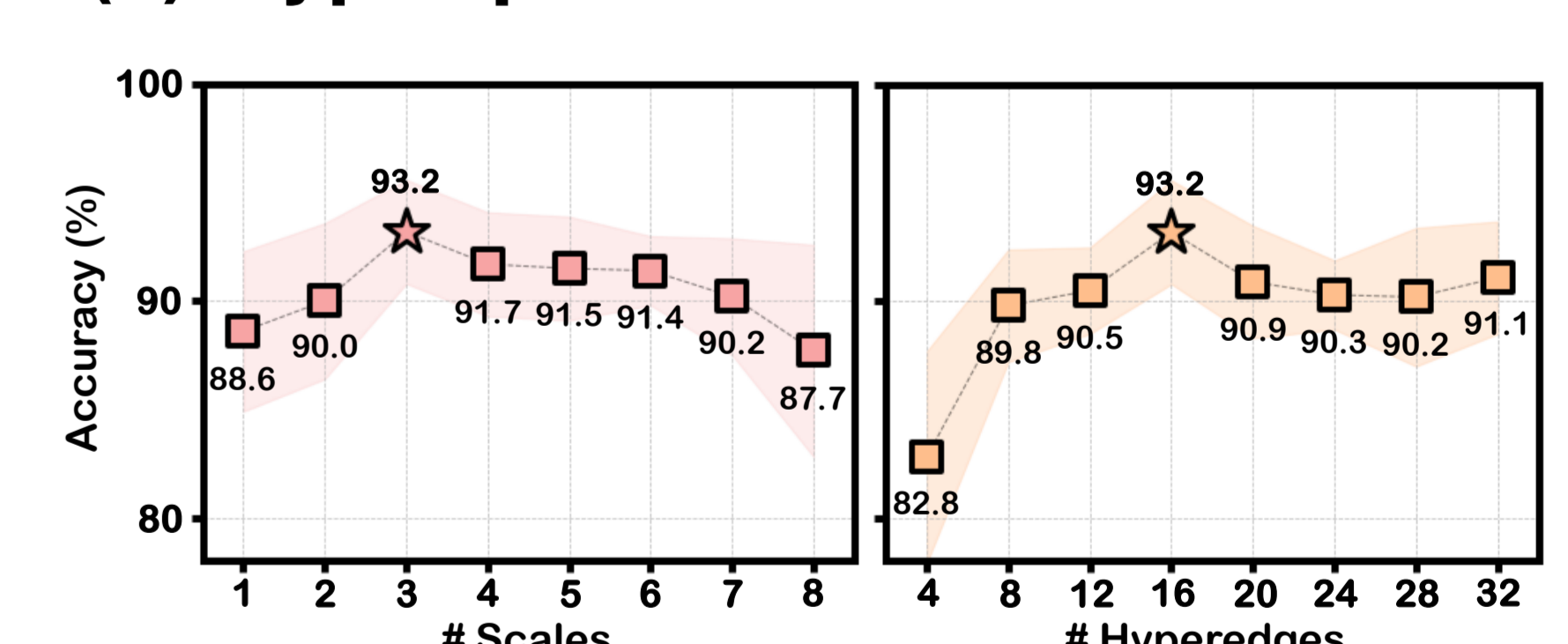


Figure: Sensitivity studies of crucial hyperparameters in MuHL on the ADNI dataset. Boxes denote the mean, with shading indicating standard deviation across folds.